

Conic Sections

1. Find the equation of the circle which touches the both axes in first quadrant and whose radius is a .
2. Show that the point (x, y) given by $x = \frac{2at}{1+t^2}$ and $y = \frac{a(1-t^2)}{1+t^2}$ lies on a circle for all real values of t such that $-1 \leq t \leq 1$ where a is any given real numbers.
3. If a circle passes through the point $(0, 0)$, $(a, 0)$, $(0, b)$ then find the coordinates of its centre.
4. Find the equation of the circle which touches x -axis and whose centre is $(1, 2)$.
5. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.
[Hint: Distance between given parallel lines gives the diameter of the circle.]
6. Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant.
[Hint: Let a be the radius of the circle, then $(-a, -a)$ will be centre and perpendicular distance from the centre to the given line gives the radius of the circle.]
7. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is $(3, 4)$, then find the coordinate of the other end of the diameter.
8. Find the equation of the circle having $(1, -2)$ as its centre and passing through $3x + y = 14$, $2x + 5y = 18$
9. If the line $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$, then find the value of k .
[Hint: Equate perpendicular distance from the centre of the circle to its radius].
10. Find the equation of a circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and has double of its area.
[Hint: concentric circles have the same centre.]
11. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.
12. Given the ellipse with equation $9x^2 + 25y^2 = 225$, find the eccentricity and foci.
13. If the eccentricity of an ellipse is $\frac{5}{8}$ and the distance between its foci is 10, then find latus rectum of the ellipse.
14. Find the equation of ellipse whose eccentricity is $\frac{2}{3}$, latus rectum is 5 and the

centre is $(0, 0)$.

15. Find the distance between the directrices of the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$.
16. Find the coordinates of a point on the parabola $y^2 = 8x$ whose focal distance is 4.
17. Find the length of the line-segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola where the line-segment makes an angle θ to the x -axis.
18. If the points $(0, 4)$ and $(0, 2)$ are respectively the vertex and focus of a parabola, then find the equation of the parabola.
19. If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$ then find the value of m .
[Hint: Solving the equation of line and parabola, we obtain a quadratic equation and then apply the tangency condition giving the value of m].
20. If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then obtain the equation of the hyperbola.
21. Find the eccentricity of the hyperbola $9y^2 - 4x^2 = 36$.
22. Find the equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at $(\pm 2, 0)$.

23. If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units, then obtain the equation of the circle.
24. Find the equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$.
25. Find the equation of a circle whose centre is (3, -1) and which cuts off a chord of length 6 units on the line $2x - 5y + 18 = 0$.
- [Hint: To determine the radius of the circle, find the perpendicular distance from the centre to the given line.]
26. Find the equation of a circle of radius 5 which is touching another circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at (5, 5).
27. Find the equation of a circle passing through the point (7, 3) having radius 3 units and whose centre lies on the line $y = x - 1$.
28. Find the equation of each of the following parabolas
 (a) Directrix $x = 0$, focus at (6, 0) (b) Vertex at (0, 4), focus at (0, 2)
 (c) Focus at (-1, -2), directrix $x - 2y + 3 = 0$
29. Find the equation of the set of all points the sum of whose distances from the points (3, 0) and (9, 0) is 12.
30. Find the equation of the set of all points whose distance from (0, 4) are $\frac{2}{3}$ of their distance from the line $y = 9$.
31. Show that the set of all points such that the difference of their distances from (4, 0) and (-4, 0) is always equal to 2 represent a hyperbola.
32. Find the equation of the hyperbola with
 (a) Vertices $(\pm 5, 0)$, foci $(\pm 7, 0)$ (b) Vertices $(0, \pm 7)$, $e = \frac{4}{3}$
 (c) Foci $(0, \pm \sqrt{10})$, passing through (2, 3)

True or False Type Questions

33. The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 + 6x + 2y = 0$.
34. The shortest distance from the point $(2, -7)$ to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is equal to 5.

[Hint: The shortest distance is equal to the difference of the radius and the distance between the centre and the given point.]

35. If the line $lx + my = 1$ is a tangent to the circle $x^2 + y^2 = a^2$, then the point (l, m) lies on a circle.

[Hint: Use that distance from the centre of the circle to the given line is equal to radius of the circle.]

36. The point $(1, 2)$ lies inside the circle $x^2 + y^2 - 2x + 6y + 1 = 0$.

37. The line $lx + my + n = 0$ will touch the parabola $y^2 = 4ax$ if $ln = am^2$.

38. If P is a point on the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ whose foci are S and S', then $PS + PS' = 8$.

39. The line $2x + 3y = 12$ touches the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 2$ at the point $(3, 2)$.

40. The locus of the point of intersection of lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different value of k is a hyperbola whose eccentricity is 2.

[Hint: Eliminate k between the given equations]